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CALCULATING EQUIVALENT CIRCUITS OF A CAVITY RESONATOR

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A method of calculating equivalent circuits of a cavity resonator is analyzed and improvements are proposed; calculations are made for an equivalent circuit of a cylindrical resonator in which standing H_{011} - type wave is produced.

1. General Statements on Equivalence of Circuits and Resonators

In designing radio installations with hollow resonators the attempt is often made to replace the resonator with equivalent oscillating circuits of lumped constants.

It should be noted that the idea of equivalence is not strictly defined; this indefiniteness is explained by the complicated structure of resonator fields, which are not incorporated in a well-defined manner with the lumped fields of ordinary circuit capacitance and inductance. For example, given the field distribution in a resonator producing free oscillations of frequency ω, it is possible to find the following physical quantities:

a. The magnetic energy of a resonator with volume w is

$$W_m = \frac{1}{8\pi} \int H^2 dw, \tag{1}$$

where H is the magnetic field strength in the element of volume dw.

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b. The electric energy of the resonator is

$$W_e = \frac{1}{8\pi}\int E^2 \, dw, \tag{2}$$

where E is the electric field strength and; hence the energy is shown to be equal to

$$W_m = W_e = W.$$

c. The total electric charge induced in the walls S of a resonator is

$$q = \frac{1}{4\pi}\int E_n \, dS. \tag{3}$$

d. The total magnetic flux in the resonator is

$$\Phi = \int H_n \cdot dS, \tag{4}$$

where S is the proper surface inside the resonator the form of which is determined by the type of oscillation.

e. The total voltage between the walls of the resonator calculated along a properly selected path l is:

$$V = \int E_l \, dl. \tag{5}$$

f. The total current in the resonator is

$$I = \frac{1}{4\pi}\oint H_l \, dl = \frac{1}{4\pi \cdot 3 \cdot 10^{10}}\int\left(\frac{\partial E}{\partial t}\right)_n dS \tag{6}$$

g. Since the type of oscillations in the resonator is assumed to be known, the proper wave length λ is determined by the dimensions of the resonator.

h. Once the losses in the walls of the resonator, characterized by the average power loss P, are found by one or another method [1,2,3], it is possible to calculate the quality of the resonator:

$$Q = \frac{1}{2}\,\frac{\omega W}{P} \tag{7}$$

A lumped-constant equivalent circuit of a given resonator will be characterized by capacity C_k, inductivity L_k, active resistance R_k, initial impedance Z_k, voltage U_k, and so forth

Formulating the conditions of equivalence differently and postulating that

$$W_k = \frac{C_k U_k^2}{2} = \frac{L_k I_k^2}{2} = W; \quad q_k = C_k U_k = q; \quad \Phi_k = L_k I_k = \Phi, \tag{8}$$

we find C_k, L_k, I_k, U_k.

But here there seems to be a disagreement between the natural wave length of the resonator λ and that of the circuit λ_k :

$$\lambda_k = 2\pi \cdot 3 \cdot 10^{10} \sqrt{L_k C_k} \neq \lambda, \qquad (9)$$

since the requirement of equivalence between C_k and L_k imposes an additional relationship between C_k and L_k which is not compatible with equation (8)

2. "Electrical" Equivalence

In practice requirements are limited to the minimum indispensable in operating. Often [4,5] we comply with demands which we shall provisionally call "electrical." These are connected with the following four situations:

a. The equation of electrical energies of resonator and circuit:

$$W = W_x = \frac{1}{2} C_x U_x^2. \qquad (10)$$

b. The equation of their electrical charges:

$$q = q_x = C_x U_x. \qquad (11)$$

c. The equation of their qualities:

$$Q = Q_x. \qquad (12)$$

d. The isochronism of their oscillations:

$$\lambda = \lambda_x = 2\pi \cdot 3 \cdot 10^{10} \sqrt{L_x C_x}. \qquad (13)$$

These equations permit determining L_x, C_x, U_x, after which the wave resistance of the circuit is determined as

$$\rho_x = \sqrt{\frac{L_x}{C_x}}, \qquad (14)$$

its active resistance as

$$R_x = \frac{\rho_x}{Q_x}, \qquad (15)$$

its initial impedance as

$$Z_x = \rho_x Q_x, \qquad (16)$$

the current in the circuit as

$$I_x = \frac{U_x}{\rho_x}. \qquad (17)$$

Moreover, the average loss in the circuit

$$P_x = \frac{1}{2} I_x^2 R_x = \frac{1}{2} \frac{U_x^2}{Z_x} \tag{18}$$

naturally coincides with the loss P in the resonator walls.

By this method of calculation then, we find that

$$I_x = I;\ \Phi_x = L_x I_x \neq \Phi;\ U_x \neq U, \tag{19}$$

which is explained by the heterogeneity of the fields in the resonator.

This method, however, becomes invalid if the configuration of the electric field in the resonator is such that there are no charges induced in its walls, since it is then impossible to apply condition (11). An example of this situation is a cylindrical resonator with a circular cross section in which a standing H_{OII} - type wave is produced (see below).

3. "Magnetic" Equivalence

In the cited cases it is necessary to substitute other equivalence requirements for (10) to (13). It seems expedient to introduce the following requirements which we shall provisionally call "magnetic":

a. The equation of magnetic energy of resonator and circuit:

$$W = W_y = \frac{1}{2} L_y I_y^2. \tag{20}$$

b. The equation of magnetic flux:

$$\Phi = \Phi_y = L_y I_y. \tag{21}$$

c. The equation of quality:

$$Q = Q_y. \tag{22}$$

d. The isochronism of oscillation

$$\lambda = \lambda_y = 2\pi \cdot 3 \cdot 10^{10} \sqrt{L_y C_y}. \tag{23}$$

These conditions permit determining L_y, C_y, Z_y and through them by use of equations (14) to (18), in which the index y is substituted for the index x, finding the remaining parameters of the circuit.

The following results, among others, are given by the magnetic equivalence method:

$$I_y \neq I,\ U_y = U;\ q_y = C_y U_y \neq q. \tag{24}$$

Of course, the last two relations of (24) have significance only when the magnetic equivalence is calculated for resonators which can be calculated by the electrical equivalence method also.

The "magnetic" method is obviously more general than the "electrical" method in its present form.

4. Connection Between Electrical and Magnetic Equivalences

The establishment of a relationship among the circuit parameters, calculated by both methods, permits a formal transition to the method of electrical equivalence, especially when direct utilization would be impossible.

The establishment of this relationship is due to the essential relations (19) and (24), which permit finding the values of the conversion factors for such parameters. It is important to note that these factors depend on the wave type and geometric form of the resonator, and not on its linear dimensions, as the factors depend only on the configuration of the field in the waveguide.

Let us take as an example the case of a rectangular resonator, in which a standing H_{110} wave is produced

$$E_x = E_y = H_z \equiv 0;\quad E_z, H_x, H_y \neq 0.$$

The electrical equivalence in this case [equations (10) - (13)] was calculated by Patrushev [4], who gives the parameters of an equivalent circuit as functions of the dimensions of the resonator.

Table 1. Rectangular Resonator H_{110} Wave

Physical Quantities	Resonator		Electrical Equivalent Circuit		Magnetic-Equivalent Circuit	
	Equation	Value	Equation	Value	Equation	Value
Magnetic or electric energy	(1), (2)	W	(10)	$W_x = W$	(20)	$W_y = W_x = W$
Electric charge	(3)	q	(11)	$q_x = q$	(24)	$q_y = \beta q_x = \beta q$
Magnetic flux	(4)	Φ	(19)	$\Phi_x = \beta\Phi$	(21)	$\Phi_y = \frac{1}{\beta}\Phi_x = \Phi$
Current	(6)	I	(17)	$I_x = I$	(20), (21)	$I_y = \beta I_x = \beta I$
Voltage	(5)	U	(10), (11)	$U_x = \beta U$	(17)	$U_y = \frac{1}{\beta}U_x = U$
Capacitance	—	—	(10), (11)	C_x	(23)	$C_y = \beta^2 C_x$
Inductance	—	—	(13)	L_x	(20), (21)	$L_y = \frac{1}{\beta^2} L_x$
Wave resistance	—	—	(14)	ρ_x	(14)	$\rho_y = \frac{1}{\beta^2}\rho_x$
Active resistance	—	—	(15)	R_x	(15)	$R_y = \frac{1}{\beta^2} R_x$
Initial impedance	—	—	(16)	Z_x	(16)	$Z_y = \frac{1}{\beta^2} Z_x$

Table 1 (Contd)

Physical Quantities	Resonator		Electrical Equivalent Circuit		Magnetic-Equivalent Circuit	
	Equation	Value	Equation	Value	Equation	Value
Power loss in oscillating system		P	(18)	$P_x = P$	(18)	$P_y = P_x = P$
Quality	(7)	Q	(12)	$Q_x = Q$	(22)	$Q_y = Q_x = Q$
Proper wave length	—	λ	(13)	$\lambda_x = \lambda$	(23)	$\lambda_y = \lambda_x = \lambda$

Here the author uses the magnetic equivalence method of calculation [equations (20) to (23)] and also determines the quantities directly characterizing the resonator, [equations (1) to (6)]. The equation numbers for calculating the various parameters are also included in the table.

It is evident from this table that the conversion factors connecting the parameters found by both methods, are determined when converting from a magnetic to an electrical equivalence as follows:

$$\beta = \frac{I_y}{I}; \qquad (25)$$

For the reverse conversion the following relation is used.

$$\beta = \frac{U_x}{U}. \qquad (26)$$

In the particular case under consideration $\beta = \frac{\pi^2}{16} = 0.617$. Patrushev calculated a rectangular resonator (electrical method) with wave types E_{111} and H_{111}. Application of the magnetic method gives analogous results and the conversion factor (25) for both waves is shown to be one and the same value: $\beta = \frac{\pi^3}{64}$.

This conversion factor was accidentally omitted in the last term of equation (28) in Patrushev's work.

5. Calculating Equivalent Circuit of a Cylindrical Resonator

Applying the magnetic equivalence method to a cylindrical resonator with a circular cross section (radius a and height h), let us place the origin of the cylindrical coordinates (ρ, φ, ζ) at the center of a base, setting the ζ-axis along the cylinder axis.

As we know [6], one of the simplest types of standing waves in such a cylinder is a H_{011} wave of frequency ω, the field of which is characterized by the following amplitudes:

$$\left.\begin{aligned} E_\varphi &= A J_1(\rho D) \sin q\zeta \\ H_\rho &= \frac{q}{k} A J_1(\rho D) \cos q\zeta \\ H_\zeta &= \frac{D}{k} A J_0(\rho D) \sin q\zeta \\ E_\zeta &= E_\rho = H_\varphi \equiv 0 \end{aligned}\right\}. \qquad (27)$$

The following notation is used here:

$$k=\frac{\omega}{c}=\frac{2\pi}{\lambda}=\sqrt{D^2+q^2},\quad q=\frac{\pi}{h},\quad c=3\cdot10^{10}\,\frac{cm}{sec},$$

(28)

J_0 and J_1 are Bessel functions of the first type; A is a constant characterizing the intensity of the resonator excitation. Since the electric field vanishes at the cylinder walls, the correct relation for the boundary conditions is

$$J_1(aD)=0,\quad D=\frac{3.832}{a}.$$

(29)

Using (1), we determine the coefficient of the magnetic energy of the resonator:

$$W=\frac{1}{8\pi}\int H^2\,dw=$$
$$=\frac{A^2h}{8D^2k^2}\int_0^{aD}\left[q^2xJ_1^2(x)+D^2xJ_0^2(x)\right]dx;$$

Since we know from the theory of Bessel functions that

$$\int_0^x xJ_n^2(x)\,dx=\frac{x^2}{2}\left[J_n^2(x)-J_{n-1}(x)J_{n+1}(x)\right];$$
$$J_{n+1}(x)=\frac{2n}{x}J_n(x)-J_{n-1}(x);$$

calculating (28) and (29), we obtain

$$W=\frac{A^2}{16}J_0^2(aD)\,a^2h.$$

(30)

Magnetic flux is determined by equation (4), and for the surface of integration it is convenient to take a circle lying in the cross section $\zeta=\frac{h}{2}$, having a radius $\rho=b$, satisfying the condition

$$J_0(bD)=0;\quad bD=2.405,$$

since beyond the limits of this circle, the sign of the magnetic flux changes. We now obtain

$$\Phi=A\frac{D}{k}\int_0^b J_0(\rho D)\,2\pi\rho\,d\rho=A\cdot\frac{bD}{aD}\cdot J_1(bD)\,a\lambda.$$

The same result is obtained by calculating the flux of the radial component of the field through a cylindrical surface of radius $\rho=b$ and height

$$\zeta=\frac{h}{2}.$$

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Equation (6) permits writing the total current in the resonator:

$$I = \frac{\omega}{4\pi \cdot 3 \cdot 10^{10}} \int_0^a \int_0^h E_\varphi \, d\rho \, d\zeta = \frac{A}{\pi a D} [1 - J_0(aD)] \frac{ah}{\lambda}.$$

Similar results are also obtained when the current is calculated as the circulation of a magnetic field in a circuit limiting half of the plane axial cross section of the resonator.

Then using equations (14) to (18) and (20) to (24), it is easy to determine all parameters of the "magnetic" equivalent circuit, if the quality Q, calculated below, is known.

Finally, equation (25) permits determining the conversion factor which is independent of the dimensions of the resonator; if necessary, it also permits conversion to a circuit which is the "electrical" equivalent of the resonator, by using the relations of Table 1. We obtain

$$\theta = \frac{I_y}{I} = \frac{\pi}{8} \frac{(aD)^2}{bD} \frac{J_0^2(aD)}{J_1(bD)} \frac{1}{1 - J_0(aD)} = 0.534.$$

Table 2 gives the physical quantities of a resonator calculated according to this method; the table also gives the "magnetic" equivalent to circuit for the resonator.

It is interesting to observe that the voltage U_y proves to be equal to the "resonator voltage" U, calculated as the circulation of the electric field strength vector in the region of radius $\rho = b$ (corresponding to the field maximum) and lying in the cross section $\zeta = 0.5h$:

$$U = \int_0^{2\pi b} E_\varphi \, dl = 2\pi A \frac{bD}{aD} J_1(bD) a = U_y. \tag{31}$$

A similar equation for field strengths [equation (24)] is obtained by the method of magnetic equivalence.

6. Calculating the Quality of a Resonator

The quality of a resonator is easily determined by the "surface effect" method [1].

The value of energy introduced into equation (7) is given in equation (30).

To determine the average power loss in resonator walls, it is necessary to take into account the fact that the radial component of a magnetic field vanishes on the lateral surface of a resonator and that the axial component is the value at the bases. Hence, we obtain for the power loss the simple expression:

$$P = \frac{1}{16\pi} \sqrt{\frac{3 \cdot 10^{10}}{\lambda \gamma}} \left[\int_0^h \int_0^{2\pi} H_\zeta^2 \, d\zeta \, a \, d\varphi + 2 \int_0^a \int_0^{2\pi} H_\rho^2 \, d\rho \cdot \rho \, d\varphi \right] =$$

$$= \frac{1}{32} \sqrt{\frac{3 \cdot 10^{10}}{\gamma}} \lambda^{3/2} A^2 J_0^2(aD) a^2 h \left[\frac{(aD)^2}{2\pi^2 a^3} + \frac{1}{h^3} \right], \tag{32}$$

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where the first integral is taken at $\rho=a$, the second at $\zeta=0$, σ is the electroconductivity of the material of the resonator walls, expressed in the CGS system.

Substituting (30) and (32) in equation (7), we shall find:

$$Q=2\pi\sqrt{3\cdot10^{10}}\,\gamma\frac{1}{\lambda^{5/2}}\left[\frac{(aD)^2}{2\pi^2a^3}+\frac{1}{h^3}\right]^{-1}.$$

If the resonator is made of copper ($\sigma=5.71 \times 10^{-4}$), we shall obtain

$$Q=\frac{2.6\cdot10^4}{\lambda^{5/2}}\left[\frac{0.744}{a^3}+\frac{1}{h^3}\right]^{-1}.$$

This magnetic equivalence method, necessary in calculating resonators whose walls do not carry charges, may also be used in other cases, since finding the magnetic flux and the current for E-type waves, where the magnetic field is determined by only two components, is simpler than calculating the charge and voltage.

Table 2. Cylindrical Resonator, H_{011} Wave

Physical Magnitude	Resonator		Magnetic Equivalence Circuit		
	Equation	Value	Equation	Value	Units
Magnetic Energy	(1)	$W=\frac{1}{16}A^2a^2hJ_0^2(aD)$	(20)	$W_y=W=1.015\cdot10^{-2}A^2a^2h$	CGS
Magnetic Flux	(4)	$\Phi=A\frac{bD}{aD}J_1(bD)a\lambda$	(21)	$\Phi_y=\Phi=0.326\,Aa\lambda$	CGSM
Current	(6)	$I=\frac{A}{\pi aD}[1-J_0(bD)]\frac{\lambda}{2}$	(20) (21)	$I_y=cI=6.25\cdot10^{-2}\frac{Aah}{\lambda}$	CGSM
Voltage	—	—	(17)	$U_y=2\pi\frac{bD}{aD}J_1(bD)Aa=2.148\,Aa$	CGSE
Capacitance	—	—	(23)	$C_y=\frac{1}{32\pi^2}\left(\frac{aD}{bD}\right)^2\frac{J_0^2(aD)}{J_1^2(aD)}h=4.84\cdot10^{-3}h$	CGSE
Inductance	—	—	(20) (21)	$L_y=8\left(\frac{bD}{aD}\right)^2\frac{J_1^2(bD)}{J_0^2(aD)}\frac{\lambda}{h}=5.235\frac{\lambda^2}{h}$	CGSM
Wave Resistance	—	—	(14) (23)	$\rho_y=480\pi\left(\frac{bD}{aD}\right)^2\frac{J_1^2(bD)}{J_0^2(aD)}\frac{\lambda}{h}=987\frac{\lambda}{h}$	om
Natural Wave Length	(28)	$\lambda=2\pi\left[\left(\frac{3.83}{a}\right)^2+\left(\frac{\pi}{h}\right)^2\right]^{-1/2}$		$\lambda_y=\lambda$	cm
Quality	(7) (32)	$Q=2\pi\sqrt{3\cdot10^{10}}\,\gamma\lambda^{-5/2}\left[\frac{(aD)^2}{2\pi^2a^3}+\frac{1}{h^3}\right]^{-1}$	(22)	$Q_y=Q$	—
Electric Charge	—	—	(24)	$q_y=\frac{A}{16\pi}\frac{aD}{bD}\frac{J_0^2(aD)}{J_1(bD)}ah=9.91\cdot10^{-3}hA$	CGSE

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